# PERIDYNAMIC FAILURE MODELING

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*Introduction*

In the peridynamics theory, failure modes such as damage or crack nucleation, coalescence, and propagation are associated with bond breakage between material points.

*Critical Stretch*

A bond can break if its stretch, $s$, exceeds a critical stretch, $s_0$. The critical stretch can be obtained experimentally. Furthermore, Silling and Askari [2] represented a formulation to calculate the critical stretch by comparing the fracture energy of the material, $G_0$, with the total energy required to generate a fracture surface within the horizon of a material point. The simplest way to introduce failure into peridynamic formulation is to modify the weight function, $\omega(|\boldsymbol{\xi}|)$, introducing additional term to capture the bond breakage as follows:

$$\bar{\omega}(|\boldsymbol{\xi}|) = \begin{cases} \omega(|\boldsymbol{\xi}|) & s < s_0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

*Critical Energy Density*

A failure criterion for breaking the physical bonds between material points in the NOSB-PD formulation was proposed by Foster et al. [37]. This criterion gives the threshold (or critical) energy density, $w_0$, such that a bond breakage occurs when the amount of energy density stored in a bond, $w_{\boldsymbol{\xi}}$, reaches the threshold. As a consequence, there is no physical interaction between two material points. The energy density stored in a bond is defined as

$$w_{\boldsymbol{\xi}} = \int_{\mathbf{0}}^{\boldsymbol{\eta}} \{\mathbf{T}[\mathbf{x}]\langle\mathbf{x}' - \mathbf{x}\rangle - \mathbf{T}[\mathbf{x}']\langle\mathbf{x} - \mathbf{x}'\rangle\} \cdot \mathrm{d}\boldsymbol{\eta} \tag{2}$$

which resembles the well-known work or conservation of energy equation where the integration of forces acting on a body over the travel distance (or path length) gives the work done by the external force. Noting that $\boldsymbol{\eta}$ defines the relative displacement vector and is computed by $\mathbf{u}' - \mathbf{u}$. The critical energy density can be defined in terms of material properties and has been experimentally determined for a 2D body as follows [37]:

$$w_0 = \frac{3G_f}{h\delta^3} \tag{2}$$

where the energy release rate, $G_f$, represents the energy required to open a new fracture surface of a unit area. In this study, the weight function is modified as follows to implement the failure criterion:

$$\bar{\omega}(|\boldsymbol{\xi}|) = \begin{cases} \omega(|\boldsymbol{\xi}|) & w_{\boldsymbol{\xi}} < w_0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

*Classical Damage Models*

NOSB-PD can adopt constitutive relationship and materials models conventionally used in the classical continuum mechanics formulation. In this context, damage models from classical mechanics can be used. In doing so, first a damage parameter, D, is defined for each material point based on the damage model. The damage parameter is between 0 and 1, where 0 stands for not damaged point and 1 stands for fully damaged point. The bond between two points, $\mathbf{x}$ and $\mathbf{x}'$ is broken if $D(\mathbf{x}) = 0$ and $D(\mathbf{x}') = 0$.

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